Assignment MATHEMATICS

Class : XII **Summer Vacations Assignment – 1** **Topic : Methods of Differentiation**

1. **Differentiate** $sin\, x^2 w.r.t. x^2$

2. **Find the derivative of** $sin(sin\, x^2)$.

3. **If** $y=\sqrt{\frac{1-x}{1+x}}$ **; find** $\frac{dy}{dx}$. **Hence prove that** $(1-x^2)\frac{dy}{dx}+y=0$.

4. **If** $x=\cos t+\log\tan\frac{t}{2}, y=\sin t$, **find the value of** $\frac{d^2y}{dt^2}$ **and** $\frac{d^2y}{dx^2}$ **at** $t=\frac{\pi}{4}$.

5. **Differentiate** $2^{cos^2 x}$ **with respect to x.**

6. **If** $y=tan^{-1}(e^x)$, **then find the value of** $\frac{dy}{dx}$ **at x=0**

7. **If** $f(x)=\frac{x^{100}}{100}+\frac{x^{99}}{99}+\frac{x^{98}}{98}+......x+1,$ **show that** $f'(1)=100f'(0)$

8. **Differentiate :** $\left(e^{x\log a}+e^{a\log x}+e^{a\log a}\right)$ **w.r. to x.**

9. **Find** $\frac{dy}{dx}$, **if** $xy=e^{x-y}$.

10. **If** $f(x)=x^4+4x^3-6x+2\sqrt{x}$ **find** $f''(x)$ **at x=1.**

11. **If** $e^y(x+1)=1,$ **show that** $\frac{dy}{dx}=-e^y$

12. **If** $y=\sqrt{\frac{1-x}{1+x}}$, **Prove that** $(1-x^2)\frac{dy}{dx}+y=0$

13. **If** $y=(\log x)^2$, **then prove that** $x^2y^{11}+xy^1=2$.

14. **If** $y=\cos^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$, **show that** $\frac{dy}{dx}=-\frac{2^{x+1}.\log_e 2}{1+4^x}$.

15. **If** $y=(\cot^{-1}x)^2$, **prove that** $y_2(x^2+1)^2+2x(x^2+1)y_1=2$ **where** y_2 **&** y_1 **are respectively second & first derivative of y with respect to x.**

16. **If** $x^p y^q=(x+y)^{p+q}$ **prove** $\frac{dy}{dx}=\frac{y}{x}$

17. **If** $x\sqrt{1+y}+y\sqrt{1+x}=0$. **Prove that** $\frac{dy}{dx}=\frac{-1}{(x+1)^2}$

18. **If** $(\cos x)^y=(\sin y)^x$, **find** $\frac{dy}{dx}$.

19. **If** $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right), x\neq 0,$ **find** $\frac{dy}{dx}$.

20. **If** $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y),$ **then prove that** $\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

Assignment MATHEMATICS

Class : XII | **Summer Vacations Assignment – 2** | **Topic : Application of Derivatives**

1. **Find maximum value of** $\sin x + \cos x$

2. $f(x) = 4x^2 + 2x + 1,$ **then find out the minimum value of** $f(x)$**.**

3. **Find the slope of the tangent to the curve** $y = 3x^4 - 4x$ **at** $x = 1.$

4. **Without using derivatives, show that the function** $f(x) = 2x + 5$ **is strictly increasing on R.**

5. **Find** $\frac{dy}{dx}$ **if** $y = at^2$**,** $t = \frac{x}{2a}$

6. **Find slope of tangent of** $y = x^3 + x^2 + x$ **at** $x = 1$

7. **The radius of a circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference ?**

8. **An edge of a variable cube is increasing at the rate of 10 cm per second. How fast is the volume of the cube increasing when the edge is 5 cm long ?**

9. **Find the point at which the tangent to the curve** $y = \sqrt{4x - 3} - 1$ **has its slope** $\frac{2}{3}$**.**

10. **Find slope of tangent and normal at x=0,** $y = 2x^2 + 3\sin x$

11. **Find interval in which function is stricly increasing** $y = x^2 + 2x - 5$

12. **Final local maxima or local minima of** $f(x) = e^{x+3}$**, if any.**

13. **Find interval in which** $y = (\sin x + \cos x)$ **is strictly increasing,** $0 \le x \le 2\pi$

14. **Find the point at which the tangent to the curve** $y = \left(\sqrt{4x - 1} - 1\right)$ **has its slope** $\left(2/3\right)$

15. Write inc as dec. interval of $f(x) = \sin 3x \left[0, \frac{\pi}{2}\right]$

16. Strictly decreasing for $x > -\frac{3}{2}$ and strictly increasing for $x < -\frac{3}{2}$.

17. Find approximate value of $(26)^{\frac{1}{3}}$ by differential.

18. Find absolute maximum and minimum value of $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 1$ in [1, 4]

19. Find approximate value of $\sqrt{49.5}$

20. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

21. Show that the semi vertical angle of the right circular cone of given total surface area and max. volume is $sin^{-1}\frac{1}{3}$

22. Show that the semi-vertical angle of the right circular cone of given total surface area and maximum volume is $sin^{-1}\frac{1}{3}$.

23 A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the rectangle so that it may produce the largest area of the window.

24. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volumeof the sphere.

25. A point on the hypotenuse of a right-triangle is at a distance 'a' and 'b' from the sides of the triangles. Show that minimum length of the hypotenuse is $\left(a^{2/3} + b^{2/3}\right)^{3/2}$

Assignment MATHEMATICS

Class : XII | Summer Vacations Assignment – 3 | Topic : Vector

1. Find the projection of $\vec{a}=\hat{i}+3\hat{j}+7\hat{k}$ on the vector $7\hat{i}-\hat{j}+8\hat{k}$
2. Find a unit vector in the direction of $\vec{a}=2\hat{i}-3\hat{j}+6\hat{k}$
3. Find the projection of the vector $\hat{i}-\hat{j}$ on the vector $\hat{i}+\hat{j}$
4. Write a vector of magnitude 15 units in the direction of vector $\hat{i}-2\hat{j}+2\hat{k}$.
5. Find magnitude of $\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}$?
6. Find a unit vector in the direction of the vector $\vec{a}=\hat{i}-2\hat{j}$ has magnitude 7 units.
7. If $|\vec{a}|=\sqrt{3}, |\vec{b}|=2$ and $\vec{a}.\vec{b}=3$, find the angle between $\vec{a}$ and $\vec{b}$
8. Find projection of a vector of direction ratio (3, 4, 5) to the vector of direction ratio (4, 3, 5)
9. If $\vec{a}=7\hat{i}+\hat{j}-4\hat{k}$ and $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$ then find the projection of $\vec{a}$ on $\vec{b}$
10.. If $|\vec{a}|=13, |\vec{b}|=5$ and $\vec{a}.\vec{b}=60$ then find $|\vec{a}\times\vec{b}|$
11. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively and $\vec{a}\bullet\vec{b}=\sqrt{6}$
12. The position vectors $\vec{a},\vec{b},\vec{c}$ of three given points satisfy the relation $4\vec{a}-9\vec{b}+5\vec{c}=0$. Prove that three points are collinear.
13. If $\vec{a}=\hat{i}+2\hat{j}-\hat{k}$, $\vec{b}=3\hat{i}+\hat{j}-5\hat{k}$ then find a unit vector in the direction of $\vec{a}-\vec{b}$
14. If $\vec{a}$ is a unit vector such that $\vec{a}\times\hat{i}=\hat{j},$ find $\vec{a}.\hat{i}$
15. Find area of triangle formed by O, A and B where $\overrightarrow{OA}=\hat{i}+2\hat{j}+3\hat{k}$ and $\overrightarrow{OB}=-3\hat{i}-2\hat{j}+\hat{k}$
16. Find area of triangle formed by O, A and B where $\overrightarrow{OA}=2\hat{i}+4\hat{j}+5\hat{k}$ and $\overrightarrow{OB}=-3\hat{i}-2\hat{j}+\hat{k}$
17. Express the vector $\vec{a}=5\hat{i}-2\hat{j}+5\hat{k}$ as sum of two vectors such that one is parallel to the vector $\vec{b}=3\hat{i}+\hat{k}$ and other is perpendicular to $\vec{b}$.
18. Find the values of x and y if the vectors $3\hat{i}+x\hat{j}-\hat{k}$ and $2\hat{i}+\hat{j}+y\hat{k}$ are mutually perpendicular vectors of equal magnitude.
19. Three vectors $\vec{a},\vec{b},\vec{c}$ are given which are mutually perpendicular and $|\vec{a}|=|\vec{b}|=|\vec{c}|$.

Find the angle between $\vec{a}$ and $\left(\vec{a}+\vec{b}+\vec{c}\right)$ and show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.

20. Show that $|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$, for any two non-zero vectors $\vec{a}$ and $\vec{b}$.

21. Let $\vec{a}$ and $\vec{b}$ be unit vectors and θ is the angle between them. Show that :

(a) $\sin\frac{\theta}{2}=\frac{1}{2}\left|\vec{a}-\vec{b}\right|$ (b) $\cos\frac{\theta}{2}=\frac{1}{2}\left|\vec{a}+\vec{b}\right|$

22. Find the scalar components of a unit which is perpendicular to the vectors $\hat{i}+2\hat{j}-\hat{k}$ and $3\hat{i}-\hat{j}+2\hat{k}$.

23. If a unit vector, $\vec{a}$ makes angle $\frac{\pi}{4}$ with $\hat{i}$; $\frac{\pi}{3}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find the components of $\vec{a}$ and the angle θ.

24. Show that the four points having position vectors $6\hat{i}-7\hat{j}, 16\hat{i}-19\hat{j}-4\hat{k}, 3\hat{j}-6\hat{k}$ and $2\hat{i}-5\hat{j}+10\hat{k}$ are coplanar.

25. Prove that $\left[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}\right]=2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$.

Class : XII Summer Vacations Assignment – 4 Topic : 3 – D Straight Line & Planes

1. Write the direction cosines of a line parallel to the line $\frac{3-x}{3}=\frac{y+2}{-2}=\frac{z+2}{6}$
2. Find the direction cosines of the line passing through the following points : $(-2, 4,-5),(1, 2,3)$
3. Write the angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}, z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$
4. Write direction cosine of a vector whose direction ratio are 2, 3,4.
5. Write equation of line $\vec{r}=2\hat{i}+\hat{j}+\lambda\left(5\hat{i}-2\hat{j}+\hat{k}\right)$ in Cartesian form.
6. Find the direction ratio of the line given by $\frac{x-2}{4}=\frac{3-y}{1}=\frac{z}{4}$
7. Find the unit normal vector of the plane. $\vec{r}.\left(3\hat{i}+4\hat{j}\right)=5$
8. Find the position vector of the mid-point of the vector joining the pointsP(2, –1, –5) and Q(4, 5, –3)
9. Find the angle between the lines $\vec{r}_1=2\hat{i}-3\hat{j}+4\hat{k}+\lambda\left(\hat{i}+2\hat{j}+2\hat{k}\right)$ and $\vec{r}_2=2\hat{i}-5\hat{k}+\hat{j}+\mu\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$
10. Find the ratio in which the plane $x+y+z=2$ divides the line segment joining the points A(2,3,1) and B(–4,2,3).
11. If $\vec{r}$ is any vector in space, then show that $\vec{r}=\left(\vec{r}\bullet\hat{i}\right)\hat{i}+\left(\vec{r}\bullet\hat{j}\right)\hat{j}+\left(\vec{r}\bullet\hat{k}\right)\hat{k}$
12. Prove that $\left[\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}\right]=0$
13. If O be the origin and coordinates of P be (1, 2, –3), then find the equation of the plane through P and perpendicular to OP.
14. Show that planes $2x+y-z=5$ and $3x-2y+4z=-4$ are perpendicular
15. Find the equation of line in vector form passing through the points whose p.v. are $\hat{i}-2\hat{j}+\hat{k}$ and $-2\hat{j}+3\hat{k}$
16. Find the value of λ so that four points with the P.V., $-\hat{j}+\hat{k}$, $2\hat{i}-\hat{j}-\hat{k}$, $\hat{i}+\lambda\hat{j}+\hat{k}$ and $3\hat{j}+3\hat{k}$ are coplanar..

17. Find the vector equation of the line which passes through the point (1,–2,3) and parallel to the vector $\hat{i}+2\hat{j}-3\hat{k}$. Deduce the corresponding equation in cartesian form.

18. Find the coordinates of the point where the line through (3, –4, –5) and (2, –3, 1) crosses the plane $2x+y+z=7$

19. Find the distance of the point (2, 2, –1) from the plane $x+2y-z=1$ measured parallel to the line $\frac{x+1}{2}=\frac{y+1}{2}=\frac{2}{3}$

20. Find the equation of the plane passing through the point (–1, 3, 2) and perpendicular to each of the planes $x+2y+3z=5$ and $3x+3y+z=0$

21. Find the shortest distance between the line given as $\frac{x-5}{3}=\frac{y-7}{-16}=\frac{z-3}{7}$, $\frac{x-3}{3}=\frac{y+3}{8}=\frac{z-25}{-5}$

22. Find the co-ordinate of the foot of perpendicular drawn from the origin to the plane $3x-4y+5z-8=0$

23. Find the equation of the plane passing through the points A, B, C with position vector $\hat{i}+\hat{j}-\hat{k}, 2\hat{j}-\hat{k}, 3\hat{i}-\hat{k}$ respectively.

24. Find the S.D. between the lines given by $\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$ and $\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}$

25. Find the co-ordinate of the foot of perpendicular drawn from the origin to the plane $3x-4y+5z-8=0$

26. Find the distance of the point (1, –2, 3) from the plane $x-y+z=5$ measured along a line parallel to $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$

27. Find the equation of the plane passing through the line of intersection of the planes $2x+3y-z+1=0$ and $x+y-2z+3=0$ and perpendicular to the plane $3x-y-2z-4=0$. Also find the inclination of this plane with the xy–plane.

28. Find the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$

29. Find the image of (1, 2, 3) in the line $\vec{r}=6\hat{i}+7\hat{j}+7\hat{k}+\lambda\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$

30. Find the equation of the plane passing through (1, 1, –1) and perpendicular to each of the planes $x+2y+3z-7=0$ and $2x-3y+4z=0$

Answer Key

Class - XII Assignment -1 (Methods of Differentiation)

1.	$\cos x^2$	8.	$a^x \log a + ax^{a-1}$
2.	$2x.\cos\left(\sin x^2\right).\cos x^2$	9.	$\frac{yx-y}{xy+x}$
4.	$\left(-\frac{1}{\sqrt{2}}, 2\sqrt{2}\right)$	10.	$\frac{71}{2}$
5.	$-2\sin^2 x . 2\cos^2 x . \log_2$	18.	$\frac{\log\sin y + y\tan x}{\log\cos x - x\cot y}$
6.	$\frac{1}{2}$	19.	$\frac{x}{\sqrt{1-x^4}}$

Class - XII Assignment -2 (Application of Derivatives)

1.	$\sqrt{2}$	13.	$\left(0, \frac{\pi}{4}\right), \left(\frac{5\pi}{4}, 2\pi\right)$
2.	$\frac{3}{4}$	14.	$\left(\frac{5}{2}, 2\right)$
3.	8	15.	Strictly decreasing $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$
5.	t		Strictly increasing $\left[0, \frac{\pi}{6}\right]$,
6.	6	17.	[2.9629]
7.	1.4π cm/sec		
8.	750 cm^3/sec	18.	Absolute minimum – 63 at $x = 2$
9.	(3, 2)		Absolute maximum 257 at $x = 4$
10.	$\left(3, -\frac{1}{3}\right)$	19.	7.0357
11.	$(-1, \infty)$	23.	$4\left(6+\sqrt{3}\right), 3\left(2-\sqrt{3}\right)$
12.	No local maxima or minima		

Class - XII Assignment – 3 (Vector)

1.	$\frac{60}{\sqrt{114}}$	9.	$\frac{8}{7}$	18.	$x = \frac{-31}{12}, y = \frac{41}{12}$
2.	$\frac{\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)}{7}$	10.	25	19.	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
3.	0	11.	$\frac{\pi}{4}$	22.	<3, – 5, – 7)
4.	$5\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$	13.	$\frac{\left(-2\hat{i} + \hat{j} + 4\hat{k}\right)}{\sqrt{2}}$.	23.	$\frac{1}{\sqrt{2}}, \frac{\pi}{4}$

6. $\frac{(\hat{i}-2\hat{j})(7)}{\sqrt{5}}$ 14. 0

7. $\frac{\pi}{6}$. 15. $\sqrt{45}$

8. $\frac{49}{5\sqrt{2}}$ 16. $\frac{(\sqrt{549})}{2}$

Class - XII Assignment – 4 (3 –D Straight Line and Planes)

1. $<\frac{3}{7},-\frac{2}{7},\frac{6}{7}>$ 15. $\vec{r}=\hat{i}-2\hat{j}+\hat{k}+1(\hat{i}-2\hat{k})$

2. $<\frac{3}{\sqrt{77}},-\frac{2}{\sqrt{77}},\frac{8}{\sqrt{77}}>$ 16. $-\frac{4}{5}$

3. $\theta=\cos^{-1}\left(\frac{-3}{\sqrt{182}}\right)$ 17. $\hat{r}=\hat{i}-2\hat{j}+3\hat{k}+1(\hat{i}+2\hat{j}-3\hat{k}),\ \frac{x-1}{1}=\frac{y+2}{2}=\frac{z-3}{-3}$

4. $<\frac{2}{\sqrt{29}},\frac{3}{\sqrt{29}},\frac{4}{\sqrt{29}}>$ 18. (1, – 2, 7)

5. $\left(\frac{x-2}{5},\frac{y-1}{-2},\frac{z}{1}\right)$ 19. $\frac{4\sqrt{2}}{3}$

6. $<4,1,4>$ 20. $7x-8y+3z+25=0$

7. $\left(\frac{3\hat{i}+4\hat{j}}{5}\right)$ 21. $\frac{98}{\sqrt{29}}$

8. (3, 2, – 4) 22. $\left(\frac{24}{7},\frac{32}{7},\frac{40}{7}\right)$

9. $\theta=\cos^{-1}\left(\frac{19}{29}\right)$ 24. $\frac{8}{\sqrt{29}}$

10. 4 : 1 25. $\left(\frac{24}{7},\frac{32}{7},\frac{40}{7}\right)$

13. $x+2y-3z=14$ 27. $7x+13y+4z-9=0$

28. $\frac{1}{\sqrt{6}}$ 29. (5, 8, 15)

30. $17x+2y-7z=26$